1. Solve the following differential equations by variables separately:

(1) \( \frac{dy}{dx} = 1 - x^2 + y^2 - x^2 y^2 \)

(2) \( \frac{dy}{dx} = \log(x + 1) \)

(3) \( \frac{dy}{dx} = \frac{1 + y^2}{1 + y^2} \)

(4) \( \log \left( \frac{dy}{dx} \right) = (ax + by) \)

(5) \( \frac{dy}{dx} = 1 - x + y - xy \)

(6) solve \( \frac{dy}{dx} = y \sin 2x \), given that \( y(0) = 1 \)

(7) solve \( (1 + e^{2x}) \frac{dy}{dx} + e^x + c + y^2 \) \( dx = 0 \) given that \( y = 1 \) when \( x = 0 \)

\( e.g. \frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x} - \frac{dy}{dx} \frac{1 - y^2}{\sqrt{1 - x^2}} = 0 \)

(8) \( (x+y) \frac{dy}{dx} = 2x^3 y \)

Solve the following homogenous differential equations:

1. \( x \frac{dy}{dx} - y \frac{dx}{dy} = \sqrt{x^2 + y^2} \)

2. \( \frac{dy}{dx} = \frac{2x - y}{x + y} \)

3. \( \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \)

4. \( (x^2 + y^2) \frac{dy}{dx} - x^2 \frac{dx}{dy} = 0 \)

5. \( (x^3 - 3xy^2) \frac{dx}{dy} = (y^3 - 3x^2 y) \frac{dy}{dx} \)

6. \( \frac{dy}{dx} = \frac{y - x}{y + x} \)

7. \( x^2 \frac{dy}{dx} = 2xy + y^2 \)

8. \( x^2 \frac{dy}{dx} = 2xy + y^2 \)

9. \( y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx} \)

10. \( \frac{dy}{dx} = \frac{x^2 - y^2}{3xy} \)

Solve the following linear differential equations:

1. \( x \frac{dy}{dx} + y = x^3 \)

2. \( \frac{xdy}{dx} + y \cos x = 2 \sin^2 x \cos x \)

3. \( x^2 \frac{dy}{dx} + xy = y^2 \)

4. \( (x + 2y^3) \frac{dy}{dx} = 5 \)

5. \( \frac{xdy}{dx} - y = x^2 \)

6. \( \frac{xdy}{dx} - y = x + 1 \)

7. \( \frac{dy}{dx} + 2y = \sin x \)

8. \( \frac{dy}{dx} \frac{y}{l} = 2x^2 \cos^2 x + y = \tan x \)

9. \( \frac{dy}{dx} + y = e^{-2x} \)

10. \( \frac{dy}{dx} + y = e^{-2x} \)

Represent the following families of curves by forming corresponding differential equations, where a and b are parameters:

1. \( x^2 + y^2 = 2ax \)

2. \( y = a \cos (x + b) \)

3. \( y = \sin 2x + \cos 2x \)

4. \( y = ae^{2x} + be^{-2x} \)

Finding equations of a curve whose geometrical properties are given:

Q1. The slope of curve passing through \((4, 3)\) at any point is the reciprocal of twice the ordinate at that point. Show that the curve is a parabola.

Q2. Find the equation of a curve which passes through the point \((-2, 3)\) and the slope of whose tangent at any point \((x, y)\) is \(x + y\).

Q3. A population grows at the rate of 5% per year. How long does it take for the population to double?

Q4. Find the equation of the curve which passes through the point \((1, 3)\) and whose slope at \((x, y)\) is \(y/x^2\).

Q5. Find the equation of the curve passing through origin given that the slope of the tangent to the curve at any point \((x, y)\) is equal to the sum of the coordinates of the point.